CS 383
HW 8

Due on the last day of classes: Thursday, December 12.

1. Alan Turing was interested in modeling computations rather than accepting/rejecting inputs. His TMs had no Accept state. Given an input they either halted (which is good) or ran forever. So let $\mathcal{L}_{halt}$ = {(M,w) | M is a TM that halts (whether or not in a final state) on input w} If you prefer you can write this as {m1111w | m is the encoding of a TM that halts on input w}. Show that $\mathcal{L}_{halt}$ is recursively enumerable but not recursive.

2. We showed that if a language and its complement are both RE then both are recursive. Suppose we have 3 recursively enumerable languages that are disjoint (no string is in two of them) and whose union is the set of all strings. Show that all three must be recursive.

3. Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are both recursively enumerable. Is the concatenation $\mathcal{L}_1\mathcal{L}_2$ RE? Why or why not?

4. We know $\mathcal{L}_{ne}$ is recursively enumerable but not recursive. Let $\mathcal{L}_{2ne}$ be {m | m encodes a TM that accepts at least 2 strings} Rice's Theorem says $\mathcal{L}_{2ne}$ is not recursive. Is it recursively enumerable? Why or why not?

5. Let $\mathcal{L}_{inf}$ be {m | m encodes a TM that accepts infinitely many strings}. Is $\mathcal{L}_{inf}$ RE?

6. Let $\mathcal{L}_{hippy\text{-}dippy}$ be the set of encodings of Turing Machines that accept all strings. Our friend Happy (actually, his encoding) is a member of $\mathcal{L}_{hippy\text{-}dippy}$. The complement of $\mathcal{L}_{hippy\text{-}dippy}$ is $\mathcal{L}_{skeptical}$, the set of Turing Machines for which there is at least one string the machine fails to accept. Rice's Theorem tells us that neither of these sets is Recursive.
   a. Prove that $\mathcal{L}_{hippy\text{-}dippy}$ is not Recursively Enumerable. You might try reducing the complement of the halting language from Question 1 to $\mathcal{L}_{hippy\text{-}dippy}$.
   b. Either prove that $\mathcal{L}_{skeptical}$ is Recursively Enumerable or prove it isn't.